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# The effect of surface and Coulomb interaction on the liquid–gas phase transition of finite nuclei

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## Abstract

By means of the Furnstahl, Serot and Tang's model, the effects of surface tension and Coulomb interaction on the liquid–gas phase transition for finite nuclei are investigated. A limit pressure  $p_{\text{lim}}$  above which the liquid–gas phase transition cannot take place has been found. It is found that comparing to the Coulomb interaction, the contribution of surface tension is dominate in low temperature regions. The binodal surface is also addressed.

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Since the arguments given by Müller and Serot [1] that a second order liquid–gas (L–G) phase transition will take place in a multi-components and multi-conserved charged system, much theoretical effort has been devoted for studying this problem by using different models and different treatments [2–4]. But all investigations are limited to infinite nuclear matter. It is of interest to extend this study to finite nuclei. This is the objective of this Letter.

If we consider the finite nuclei as a liquid droplet and discuss its L–G phase transition, two major effects, namely, the surface energy of the droplet and the Coulomb interaction of proton–proton must be considered. The reasons are as follows: it has been shown that the difference of the chemical potentials between proton and neutron plays an essential role

to determine the order of L–G phase transition [1,2]. For the infinite nuclear matter the chemical potentials of proton and neutron depend on the third component  $I_3$  of isospin when nucleon–nucleon– $\rho$ -meson ( $NN\rho$ ) interaction exists. In symmetric nuclear matter, the L–G phase transition is of first order because  $I_3 = 0$ . In asymmetric nuclear matter,  $I_3 \neq 0$  and then the chemical potential of neutron  $\mu_n$  does not equal to the chemical potential of proton  $\mu_p$ , a second order phase transition may take place. The Coulomb interaction cannot be taken into account because it becomes divergent in *infinite* nuclear matter. But in *finite* nuclei, the contribution of Coulomb interaction can be considered. Obviously, the chemical potential of proton  $\mu_p$  not only depends on  $I_3$ , but also on Coulomb interaction. But the later has no effect on  $\mu_n$ . The contribution of Coulomb interaction will make that the values of  $\mu_p$  and  $\mu_n$  becomes more different.

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Besides Coulomb interaction, on the other hand, the surface tension of the droplet will affect on the pressure of the liquid phase and then on the coexistence equations because the pressures of two phases must equal at the phase transition point.

To exhibit the effects of surface energy and Coulomb interaction on the L–G phase transition for finite nuclei, we employ the Furnstahl–Serot–Tang (FST) model [5–8], which has been shown to be successful to explain the properties of both infinite nuclear matter and finite nuclei. The Lagrangian density of FST model under mean field approximations reads

$$L_{\text{MFT}} = \bar{\Psi} \left[ i\gamma^\mu \partial_\mu - (M - g_s \phi_0) - g_v \gamma^0 V_0 - \frac{1}{2} g_\rho \tau_3 \gamma^0 b_0 \right] \Psi + \frac{1}{2} m_v^2 V_0^2 \left( 1 + \eta \frac{\phi_0}{S_0} \right) + \frac{1}{4!} \zeta (g_v V_0)^4 + \frac{1}{2} m_\rho^2 b_0^2 - H_q \left( 1 - \frac{\phi_0}{S_0} \right)^{4/d} \left[ \frac{1}{d} \ln \left( 1 - \frac{\phi_0}{S_0} \right) - \frac{1}{4} \right], \quad (1)$$

where  $g_s$ ,  $g_v$ ,  $g_\rho$  are, respectively, the couplings of light scalar meson  $\sigma$ , vector meson  $\omega$  and isovector meson  $\rho$  fields to the nucleon,  $\phi_0$ ,  $V_0$ ,  $b_0$  are the expectation values  $\phi_0 \equiv \langle \phi \rangle$ ,  $\langle V_\mu \rangle \equiv \delta_{\mu 0} V_0$ ,  $\langle b_{\mu 3} \rangle \equiv \delta_{\mu 0} b_0$ . The scalar fluctuation field  $\phi$  is related to  $S$  by  $S(x) = S_0 - \phi(x)$  and  $H_q$  is given by  $m_s^2 = 4H_q/(d^2 S_0^2)$ ,  $d$  is the scalar dimension. By using the standard technique of statistical mechanics, we get the thermodynamic potential  $\Omega$  as [2,9]

$$\Omega = V \left\{ H_q \left[ \left( 1 - \frac{\phi_0}{S_0} \right)^{4/d} \times \left( \frac{1}{d} \ln \left( 1 - \frac{\phi_0}{S_0} \right) - \frac{1}{4} \right) + \frac{1}{4} \right] - \frac{1}{2} m_\rho^2 b_0^2 - \frac{1}{2} \left( 1 + \eta \frac{\phi_0}{S_0} \right) m_v^2 V_0^2 - \frac{1}{4!} \zeta (g_v V_0)^4 \right\} - 2k_B T \left[ \sum_{k, \tau} \ln(1 + e^{-\beta(E^*(k) - v_\tau)}) + \sum_{k, \tau} \ln(1 + e^{-\beta(E^*(k) + v_\tau)}) \right], \quad (2)$$

where  $\beta = 1/k_B T$  and the quantity  $v_i$  ( $i = n, p$ ) is related to the usual chemical potential  $\mu_i$  by the

equations

$$v_n = \mu_n - g_v V_0 + \frac{g_\rho^2 \rho_3}{4m_\rho^2}, \quad (3)$$

$$v_p = \mu_p - g_v V_0 - \frac{g_\rho^2 \rho_3}{4m_\rho^2}, \quad (4)$$

where  $\rho_3 = \rho_p - \rho_n$ , the third component of isospin  $I_3 = (N_p - N_n)/2 = V\rho_3/2$ , and  $E^*(k) = \sqrt{M^{*2} + k^2}$  with  $M^* = M - g_s \phi_0$ . Usually, instead of  $\rho_3$ , we introduce  $\alpha = (\rho_n - \rho_p)/\rho$ , the asymmetric parameter to calculate, where  $\rho = \rho_n + \rho_p$ . Having obtained the thermodynamic potential, all other thermodynamic quantities, for example, pressure  $p$ , can be calculated. We get

$$p = -H_q \left\{ \left( 1 - \frac{\phi_0}{S_0} \right)^{4/d} \left[ \frac{1}{d} \ln \left( 1 - \frac{\phi_0}{S_0} \right) - \frac{1}{4} \right] + \frac{1}{4} \right\} + \frac{1}{2} \left( 1 + \eta \frac{\phi_0}{S_0} \right) m_v^2 V_0^2 + \frac{1}{4!} \zeta g_v^4 V_0^4 + \frac{g_\rho^2}{8m_\rho^2} \rho_3^2 + \frac{1}{3} \frac{2}{(2\pi)^3} \int d^3 k \frac{k^2}{E^*(k)} \times [n_n(k) + \bar{n}_n(k) + n_p(k) + \bar{n}_p(k)], \quad (5)$$

where

$$n_\tau(k) = \{ \exp[(E^*(k) - v_\tau)/k_B T] + 1 \}^{-1}, \quad (6)$$

$$\bar{n}_\tau(k) = \{ \exp[(E^*(k) + v_\tau)/k_B T] + 1 \}^{-1}, \quad (7)$$

( $\tau = n, p$ )

are the nucleon and anti-nucleon distributions, respectively. The neutron density  $\rho_n$  and the proton density  $\rho_p$  are given by

$$\rho_\tau = \frac{2}{(2\pi)^3} \int d^3 k [n_\tau(k) - \bar{n}_\tau(k)] \quad (\tau = n, p). \quad (8)$$

Now we are in a position to study the L–G phase transition of finite nuclei. The two-phase coexistence equations are

$$\mu'_n(T, \rho', \alpha') = \mu''_n(T, \rho'', \alpha''), \quad (9)$$

$$\mu'_p(T, \rho', \alpha') + \mu_{\text{Coul}}(\rho') = \mu''_p(T, \rho'', \alpha''), \quad (10)$$

$$p'(T, \rho', \alpha') + p'_{\text{Coul}}(\rho') + p'_{\text{surf}}(T, \rho') = p''(T, \rho'', \alpha''), \quad (11)$$

where the prime and the double prime refer to the liquid phase and gas phase, respectively. Considering the

droplet as an uniformly charged sphere, the contribution of Coulomb interaction to the chemical potential of proton and the pressure are [9–11]

$$\mu_{\text{Coul}} = \frac{6}{5} \frac{Ze^2}{R}, \quad (12)$$

$$p_{\text{Coul}}(\rho) = \frac{Z^2 e^2}{5AR} \rho, \quad (13)$$

respectively. The additional pressure provided by the surface tension of the liquid droplet is [12,13]

$$p_{\text{surf}}(T, \rho) = -2\gamma(T)/R, \quad (14)$$

where

$$\gamma(T) = (1.14 \text{ MeV fm}^{-2}) \left[ 1 + \frac{3T}{2T_c} \right] \left[ 1 - \frac{T}{T_c} \right]^{3/2}, \quad (15)$$

with  $T_c$  being the critical temperature of L–G phase transition in symmetric nuclear matter. We take the liquid droplet along the  $\beta$ -stability line, it satisfies

$$Z = 0.5A - 0.3 \times 10^{-2} A^{5/3}. \quad (16)$$

The parameters be chosen for our numerical calculations are the set  $T_1$  of FST model

$$\begin{aligned} g_s^2 &= 99.3, & g_v^2 &= 154.5, & g_\rho^2 &= 70.2, \\ m_s &= 509 \text{ MeV}, & S_0 &= 90.6 \text{ MeV}, \\ \zeta &= 0.0402, & \eta &= -0.496, & d &= 2.70. \end{aligned} \quad (17)$$

Our results are summarized in Figs. 1–6. To make our results more transparent, we neglect the Coulomb interaction and the surface effect by taking  $R \rightarrow \infty$  in Fig. 1 and the  $\mu_n, \mu_p$  isobar vs.  $\alpha$  reduce to that of infinite asymmetric matter. In this figure, we fix the temperature  $T = 10 \text{ MeV}$  and the curves a, b, c, d, e correspond to the pressure 0.06, 0.085, 0.100, 0.164 and 0.200  $\text{MeV fm}^{-3}$ , respectively. We see that the curves for lower pressures are more complicate than those of the large pressures. When  $p = 0.200 \text{ MeV fm}^{-3}$ , curve e has one branch only, but when  $p = 0.06 \text{ MeV fm}^{-3}$ , curve a has three branches. The chemical potentials isobar  $\mu_n'', \mu_p''$  vs.  $\alpha''$  given by the right hand side of Eqs. (9)–(11) for the gas phase is shown in Fig. 2 where  $T = 5 \text{ MeV}$  and  $p = 0.016 \text{ MeV fm}^{-3}$ . In fact, these curves are the same as that of infinite nuclear matter because the chemical potential and the pressure for the gas phase do not depend on the Coulomb interaction and the surface term.

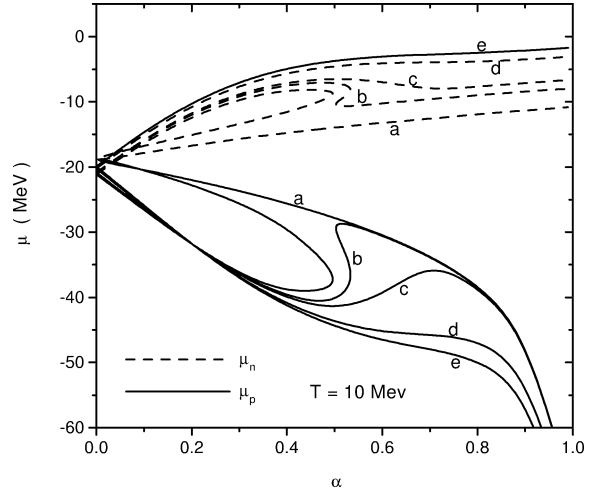


Fig. 1. The chemical isobars for infinite nuclear matter, where  $T = 10 \text{ MeV}$ , and a, b, c, d, e refer to the pressure 0.06, 0.085, 0.100, 0.164 and 0.200  $\text{MeV fm}^{-3}$ , respectively.

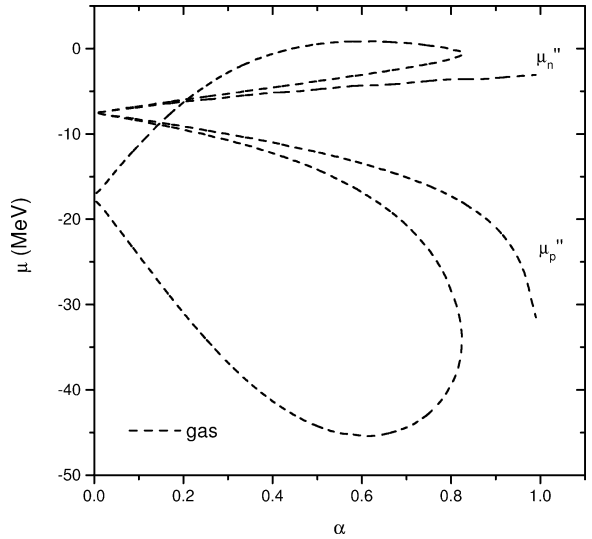


Fig. 2. The chemical isobars for the gas phase where  $T = 5 \text{ MeV}$  and  $p = 0.016 \text{ MeV fm}^{-3}$ .

We see in this case both  $\mu_p''(\alpha'')$  and  $\mu_n''(\alpha'')$  curves have three branches. The chemical isobar as a function of  $\alpha$  for the liquid phase and the gas phase are shown in Fig. 3 by solid line and dashed line, respectively, where we fixed  $T = 5 \text{ MeV}$  and  $p = 0.016 \text{ MeV fm}^{-3}$ . The dashed lines in Fig. 3 for gas phase are in fact the same curves as those of Fig. 2 except that the range of the  $\alpha$ -axis is (0.0, 0.5) instead of (0.0, 1.0). The

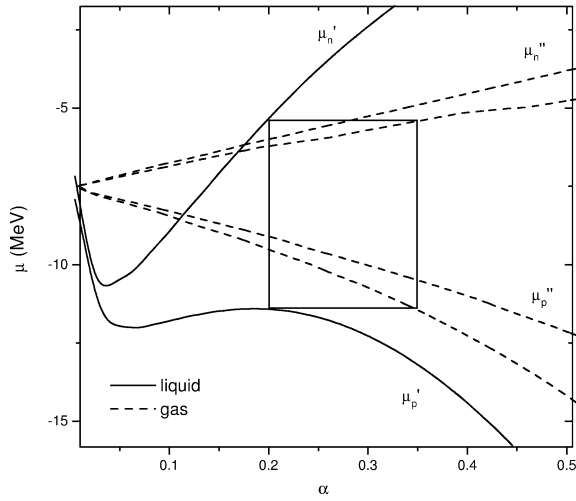


Fig. 3. The rectangle construction for two-phase equilibrium for  $T = 5$  MeV and  $p = 0.016$  MeV fm $^{-3}$ .

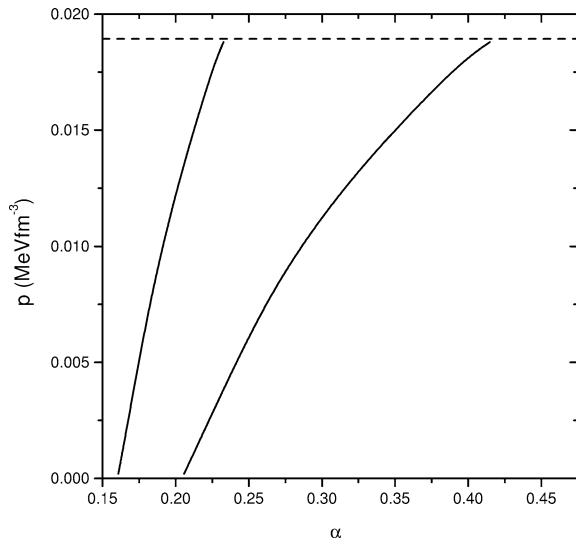


Fig. 4. The section of binodal surface for  $T = 5$  MeV.

rectangle construction [1,2] which represents for the Gibbs' conditions of Eqs. (9)–(11) for the two-phase equilibrium is also plotted in Fig. 3 by the edges of a rectangle. Due to the effect of Coulomb interaction and the surface energy, the chemical potential isobars for the gas phase and for the liquid phase are very different. We see from Fig. 3 that the  $\mu'_p(\alpha')$  and  $\mu''_n(\alpha'')$  curves for the gas phase have two branches shown by dashed lines in the regions  $0 < \alpha < 0.5$ , respectively,

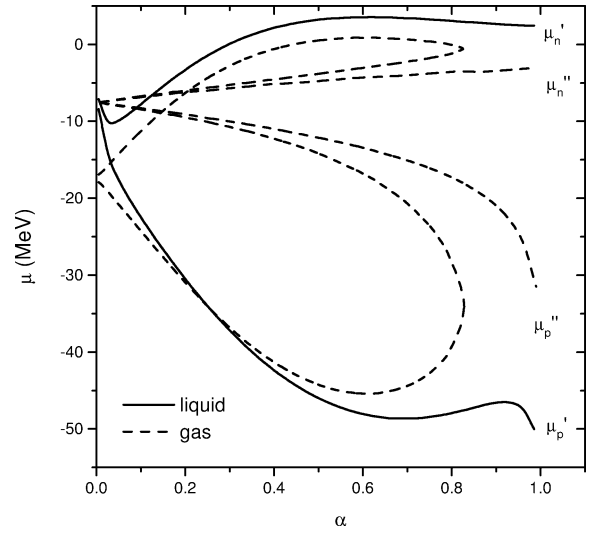


Fig. 5. The chemical isobars vs.  $\alpha$  curves with surface effect only.

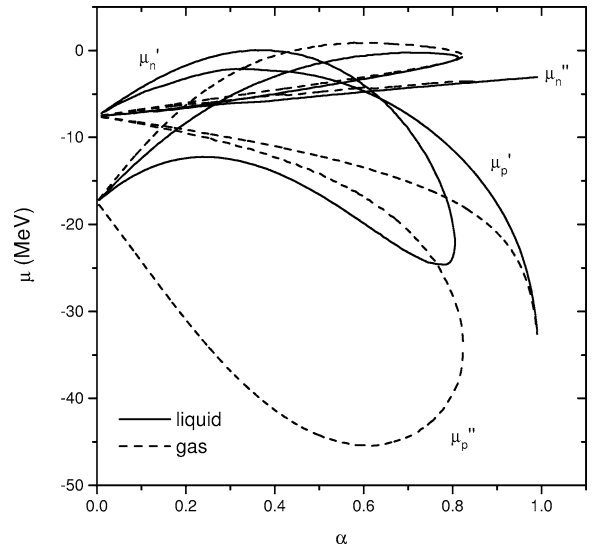


Fig. 6. The chemical isobars vs.  $\alpha$  curves with Coulomb interaction only.

but  $\mu'_p(\alpha')$   $\mu'_n(\alpha')$  for the liquid phase has one branch in this region only. This behavior is quite different from that of infinite nuclear matter in which the liquid phase and the gas phase chemical isobars  $\mu_n$  and  $\mu_p$  are shown by the same curves (for example, see curve a of Fig. 1). This new feature leads to multi-solutions for Gibbs' condition because one can find four rectangles between two branches of  $\mu''_n(\mu''_p)$  and one branch

of  $\mu'_n(\mu'_p)$ . But according to the equilibrium condition: the chemical potential of the system in equilibrium state must take the minimal value at fixed temperature and pressure [14]. Therefore, only one rectangle which corresponds to the minimum chemical potential isobar shown by solid lines in Fig. 3 refers to a stable equilibrium phase transition, and the others are all metastable states. The other three rectangles are not shown in Fig. 3.

The section of binodal surface [1,2] at finite temperature  $T = 5$  MeV is shown in Fig. 4. A limit pressure  $p_{\text{lim}} = 0.018 \text{ MeV fm}^{-3}$  above which the rectangle cannot be found and the coexisting equations Eqs. (9)–(11) have no solution has been obtained. The binodal surface will cut off at limit pressure  $p_{\text{lim}}$ . This situation is very similar to that of our previous paper [2] in which we considered the density dependence of the  $NN\rho$  coupling  $g_\rho(\rho)$ . The reason is that no matter  $g_\rho(\rho)$  or the Coulomb interaction or the surface energy, even though they change the chemical potential  $\mu_n$  and  $\mu_p$  in different fashions, they will make that the rectangle construction turns out to be disappear. In fact, this result is a reflection of the so-called Coulomb instability in finite nuclei [9–11,13,15]. The Coulomb instability of FST model has been discussed in detail by our previous paper [9] in which we found a limit temperature  $T_{\text{lim}}$  above which the coexisting equations have no solution and the L–G phase transition can not take place. The difference is that, instead of  $T_{\text{lim}}$ , we now fix temperature  $T = 5$  MeV to find the limit pressure.

Finally, we hope to compare the effects of Coulomb interaction and the surface tension on the L–G phase transition, separably. As shown in Eqs. (13) and (14), we see that, firstly, the effects of Coulomb interaction and the surface tension are opposite because of  $p_{\text{Coul}}$  and  $p_{\text{surf}}$  with opposite signs, and secondly,  $p_{\text{surf}}$  depends on temperature but  $p_{\text{Coul}}$  is independent. It means that the Coulomb interaction and the surface tension play different roles in different temperature regions. In low temperature regions  $T \leq 5 \text{ MeV} \ll T_c$ ,  $\gamma(T)$  becomes larger, and we have  $|p_{\text{surf}}| > p_{\text{Coul}}$ , the surface tension becomes dominant. To show this result clearly, we draw the chemical isobar vs.  $\alpha$  curves with the surface effect only and with the Coulomb interaction only in Fig. 5 and Fig. 6, respectively. Comparing Fig. 5, Fig. 6 and Fig. 3 we see that the curves of Fig. 5 are very similar to those of Fig. 3, but the curves of

Fig. 6 are very different. It confirms the surface effect dominates at  $T = 5$  MeV. The curves in Fig. 6 are very complicate. The reason is that we see from Eq. (11), in the phase transition process,  $p'_{\text{surf}}$  will increase  $p''$  since it has negative sign, but  $p'_{\text{Coul}}$  will decrease  $p''$ . As indicated by Fig. 1, the pressure  $p''$  becomes lower, the curve becomes more complicate.

In summary, it is shown that the surface effect and the Coulomb interaction are important for the L–G phase transition of finite nuclei. In low temperature  $T \ll T_c$  regions, the surface effect is dominate. A limit pressure  $p_{\text{lim}}$  above which the L–G phase transition cannot take place has been found. Since the critical temperature  $T_c = 14.75$  MeV, and the limit temperature  $T_{\text{lim}}$  is around 5.4–8.8 MeV for FST model [12], we come to a conclusion that the surface effect is dominate in the L–G phase transition of finite nuclei for FST model.

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